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About the Observed Asymmetry between Matter and Antimatter

Luis Grave de Peralta[1,2]

[1]Department of Physics and Astronomy, Texas Tech University, Lubbock, TX, USA
[2]Nano Tech Center, Texas Tech University, Lubbock, TX, USA
Email: luis.grave-de-peralta@ttu.edu

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Abstract

Some consequences, due to the existence of a pair of decoupled Schrödinger-like but relativistic quantum mechanics wave equations, are explored. It is shown that one equation directly describes the quantum states of a single spin-0 particle, and the other one indirectly describes the quantum states of the corresponding antiparticle. In correspondence with the matter-antimatter symmetry, for a Coulomb potential, a charge conjugation operation transforms the second equation in the first one. However, if a particle could interact with itself (gravitationally or electrically) due to the spread of its wavefunction, the C-symmetry could be broken; therefore, matter and antimatter could be distinguished. Under these assumptions, it is deducted the impossibility of the existence of particles and antiparticles with a mass larger than the Plank mass (m_P), or with the absolute value of the charge larger than the Plank charge (q_P). It is proposed the existence of primordial antimatter electrical sinks. It is also suggested that all macroscopic matter objects with a mass $m > m_P$, and all macroscopic antimatter bodies with a charge $|q| > q_P$ should not be quantum but classical objects. It is argued that these findings could explain the absence of antimatter with a complicated structure and partially explain the excess of charged matter in the known Universe.

Keywords

Relativistic Quantum Mechanics, Antimatter, Primordial Black Holes

1. Introduction

We do not live in an antimatter world but in a matter one. The observed asymmetry between matter and antimatter has two aspects. First, matter seems to be much more abundant in Nature than antimatter. Second, we are not surrounded by antimatter bodies with a complicated structure but by complicated matter

objects like matter atoms, molecules, and bulky bodies formed by them. It has been speculated that the second is a consequence of the first, and the first happens due to a yet unknown cause at the beginning of the known Universe [1] [2]. The so-called charge-parity-time reversal (CPT) symmetry justifies the current faith in such explanation. This is because we believe all laws of Physics should be CPT invariable [1] [2] [3], and because it is a general belief that the CPT symmetry implies that a matter world should be undistinguishable from the antimatter one [1] [2] [3].

In this work, it is explored a different explanation of why we are not surrounded by antimatter bodies with a complicated structure but by complicated matter bodies, and it is partially addressed the observed abundance of matter in Nature. For simplicity, only electrostatic and (Newtonian) gravitational interactions are considered in this work. The author invites the reader to start our journey in the general Grave de Peralta (gGP) equation [4] [5]:

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2\mu}\nabla^2\Psi + V\Psi. \tag{1}$$

This is a Schrödinger-like, but relativistic quantum mechanics equation, that describes the quantum states of a particle moving in a scalar potential (V) with an effective mass $\mu = \mu_+ > 0$, which depends on the parameter γ in the following way [4]:

$$\mu_+ = \frac{1+\gamma}{2}m > 0. \tag{2}$$

When $\gamma = 1$, then $\mu_+ = m$, the relativistic invariant mass of the particle; thus, Equation (1) coincides with the Schrödinger equation [6] [7]. Equation (1) is the Grave de Peralta (GP) equation when γ is the Lorentz factor of special theory of relativity [5] [8] [9] [10]. Equation (1) with $\mu = \mu_+ > 0$ has been successfully used for extending, to the relativistic domain, the non-relativistic results previously obtained using the Schrödinger equation [4] [5] [8]-[15]. Equation (1) was also used for demonstrating the impossibility of the existence of elemental quantum particles with a mass larger than the Plank mass ($m_P \approx 20\ \mu g$) [5]. In addition, it was also suggested that all matter bodies with $m > m_P$ should not be quantum but classical objects [5]. These results were obtained using the hypothesis that, due to the spread of its mass density through its wavefunction, a matter particle can gravitationally interact with itself [5] [16].

In this work, for the first time, the consequences of considering $\mu = \mu_- = -\mu_+ < 0$ are explored. First, it is shown that Equation (1), but with $\mu = \mu_-$, indirectly describes the quantum states of the antiparticle that is associated to a particle. The quantum (particle) states associated to this particle are described by Equation (1) with $\mu = \mu_+ > 0$. Second, Equation (1) is used, but with $\mu = \mu_- < 0$, in combination with the hypothesis that a particle can electrostatically interact with itself, due to the spread of its charge density through its wavefunction, for demonstrating the impossibility of the existence of elemental quantum antiparticles with the absolute value of its charge ($|q|$) larger than the Plank charge, $q_P \approx$

11e, where e is the charge of a proton. It is also suggested that all antimatter bodies with $|q| > q_P$ should not be quantum but classical objects.

Finally, it is proposed the possible existence of primordial antimatter electrical sinks, which may have formed when primordial quantum fluctuations of charged antimatter, with $|q| > q_P$, occurred at the beginning of the times. The existence of such primordial antimatter electrical sinks may partially explain the observed excess of charged matter in the known Universe. Moreover, it is suggested that we do not observe antimatter bodies with a complicated structure because antimatter atoms and molecules only can be formed when antimatter nuclei and positron clouds are quantum objects. However, they may be classical when $|q| > q_P$.

2. The gGP Equations

Like the Klein-Gordon equation, Equation (1) can be obtained, after a formal first quantization procedure, from the Lorentz-invariance of the magnitude of the four-component energy-momentum vector corresponding to a classical particle of mass m, total energy E, and three-component linear momentum $\boldsymbol{p}$, which is moving in a scalar potential V [3] [17] [18]:

$$\sqrt{\frac{(E-V)^2}{c^2}-\boldsymbol{p}^2}=mc. \tag{3}$$

In Equation (3), c is the speed of the light in vacuum, and $(E-V)$ is the sum of the kinetic energy of the particle (K) plus the energy associated to its mass (mc^2). Solving Equation (3) for $(E-V)$, we obtain two possible values of $(E-V)$ corresponding to each value of $\boldsymbol{p}$:

$$E-V=\pm\sqrt{\boldsymbol{p}^2c^2+m^2c^4}=\pm\gamma mc^2,\ \text{with}\ \gamma=\sqrt{1+\frac{p^2}{m^2c^2}}. \tag{4}$$

It is worth noting that $\gamma=1$ means the particle is at rest. In what follows, we will add the subindexes (+) and (–) to all the magnitudes related with the positive and negative values of $(E-V)$, respectively. When $(E-V)=+\gamma mc^2>0$, the relation between $(E-V)$ and K is:

$$E_+-V=+\gamma mc^2=K_++mc^2\Rightarrow K_+=(\gamma-1)mc^2\geq 0. \tag{5}$$

However, when $(E-V)=-\gamma mc^2<0$, the relation between $(E-V)$ and K should be such that $K_-=0$ when $\gamma=1$; therefore:

$$E_--V=-\gamma mc^2=K_--mc^2\Rightarrow K_-=(1-\gamma)mc^2=-K_+\leq 0. \tag{6}$$

Defining E' such that:

$$E'_\pm-V=\begin{matrix}(E_+-V)-mc^2=K_+\\(E_--V)+mc^2=K_-\end{matrix}\Rightarrow E'_\pm-V=\begin{matrix}K_+=(\gamma-1)mc^2\geq 0\\K_-=(1-\gamma)mc^2=-K_+\leq 0\end{matrix}. \tag{7}$$

And using Equation (4) for evaluating γ^2, we can obtain that K_+ and K_- are also given by the following equations:

$$\begin{aligned}&(\gamma+1)K_{\pm}=\pm(\gamma+1)\left[(\gamma-1)mc^2\right]=\pm\left(\gamma^2-1\right)mc^2=\pm\frac{\boldsymbol{p}^2}{m}\\&\Rightarrow K_{\pm}=\pm\frac{\boldsymbol{p}^2}{(1+\gamma)m}\end{aligned}\tag{8}$$

Therefore, combining Equations (7) and (8), we obtain:

$$E'_{\pm}=\pm\frac{\boldsymbol{p}^2}{(1+\gamma)m}+V.\tag{9}$$

Now, by making in Equation (9) the following formal first-quantization substitutions:

$$E'\to i\hbar\frac{\partial}{\partial t},\ \ \boldsymbol{p}\to-i\hbar\nabla.\tag{10}$$

We obtain a pair of Poirier-Grave de Peralta (PGP) equations [5]:

$$i\hbar\frac{\partial}{\partial t}\Psi_{\pm}=\mp\frac{\hbar^2}{(1+\hat{\gamma})m}\nabla^2\Psi_{\pm}+V\Psi_{\pm},\ \text{ with }\hat{\gamma}=\sqrt{1+\frac{\hat{\boldsymbol{p}}^2}{m^2c^2}}.\tag{11}$$

Readers familiar with high energy physics may recognize that the PGP equation for Ψ_+ is related with a particular case of the spinless Salpeter equation [19] [20], which is a known Lorentz-covariant alternative for the Klein-Gordon equation. Like the spinless Salpeter equation, the PGP equations are Lorentz-covariant; thus, valid relativistic quantum mechanics equations [4] [5]. Different approaches may be used for removing the operator γ in Equation (11) and substituting it by a parameter γ [4] [5]. For instance, we could use the Poveda's approach that considers γ as the average value of the operator γ in the quantum state $\Psi_{\pm}$ [5]:

$$\hat{\gamma}\to\gamma_{\pm}=\left\langle\Psi_{\pm}\left|\sqrt{1+\frac{\hat{\boldsymbol{p}}^2}{m^2c^2}}\right|\Psi_{\pm}\right\rangle.\tag{12}$$

Or consider, as originally was done by Grave de Peralta, that γ is just the relativity Lorentz factor associated to the corresponding classical particle [8]-[15]. Anyway, after substituting the operator γ by the parameter γ, we can use Equation (2) for rewriting Equation (11) as Equation (1). The case $\mu=\mu_+=(1+\gamma)m/2>0$ have been intensively studied before [4] [5] [8] [9] [10] [14] [15]:

$$i\hbar\frac{\partial}{\partial t}\Psi_{+}=-\frac{\hbar^2}{(\gamma+1)m}\nabla^2\Psi_{+}+V\Psi_{+}.\tag{13}$$

Solving Equation (13) gives the energies $E'_+=K_++V$, with $K_+>0$, and the wavefunctions Ψ_+ corresponding to a relativistic spin-0 particle of mass m, which is moving in the scalar potential V [4] [5] [8] [9] [10] [14] [15]. We will focus our attention now in the case $\mu=\mu_-=-(1+\gamma)m/2<0$:

$$i\hbar\frac{\partial}{\partial t}\Psi_{-}=\frac{\hbar^2}{(\gamma+1)m}\nabla^2\Psi_{-}+V\Psi_{-}.\tag{14}$$

Solving Equation (14) gives the energies $E'_-=K_-+V$, with $K_-<0$, and the

wavefunctions Ψ_- corresponding to the same relativistic spin-0 particle of mass m, which is moving in the scalar potential V. It should be noted that if γ is chosen as the relativity Lorentz factor associated to the corresponding classical particle, then the wavefunctions $\Psi_\mp$ are the two components of the so-called Klein-Gordon equation in the Schrödinger form [3]. When $V = -eU_C$, where U_C is the Coulomb potential $+Ze/(4\pi\varepsilon_o r)$, $r = |\boldsymbol{r}|$, and ε_o is the absolute dielectric permittivity of the vacuum, Equation (13) can be used for approximately describing (discounting the spin) the quantum states with $K > 0$ of the electron (the particle) in a Hydrogen-like atom with atomic number Z [14] [15]. On the other hand, Equation (14) can be used for approximately describing (discounting the spin) the quantum states with $K < 0$ of the electron (the particle) in a Hydrogen-like atom with atomic number Z [14] [15]. These equations are:

$$i\hbar\frac{\partial}{\partial t}\Psi_p = -\frac{\hbar^2}{(\gamma+1)m}\nabla^2\Psi_p - eU_C\Psi_p,\ U_C = +\frac{Ze}{4\pi\varepsilon_o r},\ \Psi_p = \Psi_+. \tag{15}$$

And:

$$i\hbar\frac{\partial}{\partial t}\Psi_- = \frac{\hbar^2}{(\gamma+1)m}\nabla^2\Psi_- - eU_C\Psi_-. \tag{16}$$

This pair of equations (Equations (15) and (16)) resembles the hole theory based on the Klein-Gordon and Dirac's equations [3], where there is a one-to-one relationship between Ψ_- and the positron (antiparticle) wavefunction corresponding to a quantum state with $K > 0$ (Ψ_a), which satisfies the following equation [3]:

$$i\hbar\frac{\partial}{\partial t}\Psi_a = -\frac{\hbar^2}{(\gamma+1)m}\nabla^2\Psi_a + eU_C\Psi_a. \tag{17}$$

Nevertheless, the pair of uncoupled equations (Equation (11), Equations (13) and (14), or Equations (15) and (16)) is not equivalent to the Klein-Gordon equation because, unlike the Klein-Gordon and the Dirac equations [3] [21], these pairs of equations describe processes where, first, the number of particles is constant (one) [10], and second, the kinetic energy of the particle or is always positive or is always negative. Therefore, their use is particularly useful for studying relativistic processes with these characteristics.

For instance, the following plane waves are solutions of Equations (13) and (14) for a free ($V = 0$) particle:

$$\begin{aligned}&\Psi_p = \Psi_+ = \mathrm{e}^{\frac{i}{\hbar}(\boldsymbol{p}\cdot\boldsymbol{r}-E'_+t)},\ \frac{E'_+}{mc^2} = (\gamma-1) > 0;\\ &\Psi_- = \mathrm{e}^{\frac{i}{\hbar}(\boldsymbol{p}\cdot\boldsymbol{r}-E'_-t)},\ \frac{E'_-}{mc^2} = (1-\gamma) = -\frac{E'_+}{mc^2}.\end{aligned} \tag{18}$$

Note that Ψ_- is not a solution of Equation (17) with $V = 0$ because the energy of the antiparticle should be positive and because, if the particle has a linear momentum $\boldsymbol{p}$, then the linear momentum of the antiparticle should be $-\boldsymbol{p}$. Due to the momentum conservation, if $\Psi_p = \Psi_+$ is given by Equation (18), then Ψ_a

should be given by:

$$\Psi_a = \mathrm{e}^{-\frac{i}{\hbar}\left(\boldsymbol{p}\cdot\boldsymbol{r}+E'_+t\right)},\ \frac{E'_+}{mc^2} = \left(\gamma - 1\right) > 0. \tag{19}$$

So that the free particle and antiparticle travel in opposite directions with the same energy $E'_a = E'_+ = K_+ = -E'_- = -K_-$. The wavefunction of the free antiparticle (Ψ_a) can then be obtained from Ψ_- by a charge conjugate operation [3], *i.e.*, first finding the complex conjugate of Ψ_- and then formally substituting E'_- in $(\Psi_-)^*$ by $E'_a = E'_+ = -E'_-$. It is easy to show that this is also true for the stationary states of the Equations (16) and (17) [3], *i.e.*, if:

$$\Psi_-\left(\boldsymbol{r},t\right) = \Omega_-\left(\boldsymbol{r}\right)\mathrm{e}^{-\frac{i}{\hbar}E'_-t},\ \Psi_a\left(\boldsymbol{r},t\right) = \Omega_a\left(\boldsymbol{r}\right)\mathrm{e}^{-\frac{i}{\hbar}E'_at}. \tag{20}$$

are the stationary solutions of Equations (16) and (17), or in general of Equations (13) and (14), respectively, then:

$$E'_a = -E'_- = E'_+ \ \text{ and } \ \Omega_a\left(\boldsymbol{r}\right) = \Omega^*_-\left(\boldsymbol{r}\right);\ \text{thus,}\ \left\langle\Psi_-\left|\boldsymbol{r}\right|\Psi_-\right\rangle = \left\langle\Psi_a\left|\boldsymbol{r}\right|\Psi_a\right\rangle. \tag{21}$$

Consequently, Equations (14) and (16) can be used for studying the spatial localization of the antiparticle wavefunction. This is because, due to Equation (21), the spatial localization of Ψ_- is a necessary and sufficient condition for the spatial localization of Ψ_a.

It is important to note that Ψ_+ and Ψ_- are particle (electronic) states with positive and negative kinetic energy values, respectively, while Ψ_a is an antiparticle (positronic) state with positive kinetic energy. Both particle and antiparticle interact with the same external world represented by U_C, which is the same in Equations (15), (16), and (17). Therefore, if U_C can spatially confine the electronic states with negative kinetic energy (Ψ_-), then the same U_C can confine Ψ_a. The opposite is also true, if U_C cannot confine the electronic states with negative kinetic energy (Ψ_-), then the same U_C cannot confine Ψ_a. For instance, Equation (17) describes a particle (a positron) moving with $K_{pos} > 0$ while repelled by a matter nucleus with Z protons. Unlike the electron wavefunction, Ψ_p in Equation (15), which is localized around the matter nucleus that attracts the electron, the positron wavefunction, Ψ_a in Equation (17), cannot be localized because the positron is repelled by the matter nucleus. Consequently, the wavefunction corresponding to the electron states with $K_- < 0$, Ψ_- in Equation (16), cannot be localized around the nucleus of the Hydrogen-like atom [21] [22].

Formally, Equation (17) can be obtained from Equation (16) by changing the sign of the kinetic energy term in Equation (16), and then changing the sign of the particle's charge without modifying U_C (because U_C does not depend on the particle's charge but on the external charges Ze that produce U_C). This is equivalent to the charge conjugation operation [3]; *i.e.*, Equation (17) is obtained by taking the complex conjugate of both sides of Equation (16). Finally, if the matter nucleus is substituted by the corresponding antimatter one, which concludes the charge-inversion of all the particles and antiparticles in consideration, then U_C changes of sign in Equation (17). This transforms Equation (17) in Equation

(15). Consequently, in correspondence with the C-symmetry of quantum electrodynamic [21] [22], an antimatter Hydrogen-like atom should be indistinguishable from the corresponding matter one. Note that quantum electrodynamics is independently C-symmetric and PT-symmetric; therefore, its C-symmetry implies its CPT symmetry.

3. Spatial Localization of the Wavefunctions of Particles and Antiparticles in a Coulomb Potential

An instance, illustrating why the C-symmetry in quantum electrodynamics implies that an antimatter world should be undistinguishable from the matter one, is shown in **Figure 1**. In a Hydrogen-like matter atom (**Figure 1(a)**), the electron wavefunction ($\Psi_+ = \Psi_p$) is spatially localized around the matter nucleus due to the inward acceleration of the electron, which is produced by the attractive electrical force between the electron and the matter nucleus. However, the electron (Ψ_-) and positron (Ψ_a) wavefunctions are not spatially localized around the nucleus of the Hydrogen-like matter atom.

In contrast, in a Hydrogen-like antimatter atom (**Figure 1(b)**), the positron wavefunction (Ψ_a), and thus also the electron wavefunction (Ψ_-), are both spatially localized around the antimatter nucleus due to the inward acceleration of the positron, which is produced by the attractive electrical force between the positron and the antimatter nucleus. However, the electron wavefunction ($\Psi_+ = \Psi_p$) is not spatially localized around the nucleus of the Hydrogen-like antimatter atom. Let us now describe the matter and antimatter atoms, but primarily referring to the electron (the particle) and its electronic states Ψ_+ and Ψ_- in the Hydrogen-like matter and antimatter atoms.

As illustrated in **Figure 1(a)**, the Coulombic attraction between the matter nucleus and the electron spatially localizes the wavefunction of the electron ($\Psi_+ = \Psi_p$) around the nucleus [6] [7]. In agreement with Equation (15), this happens because the negative potential energy associated to $-eU_C\Psi_+$ in Equation (15) balances the positive kinetic energy associated to [5]:

$$\hat{K}_+\Psi_+ = \frac{\hat{p}^2}{2\mu_+}\Psi_+ = -\frac{\hbar^2}{2\mu_+}\nabla^2\Psi_+, \text{ with } \mu_+ = \frac{1+\gamma}{2}m > 0. \tag{22}$$

The force associated to the nucleus-electron interaction points to the nucleus of the Hydrogen-like atom; therefore, due to the positive effective mass of the electron in the state Ψ_+ ($\mu_+ > 0$), this force produces an inward acceleration on the electron (in the state Ψ_+) that tends to spatially localize Ψ_+. In opposition, the kinetic energy of the electron tends to spread Ψ_+.

In contrast with what occurs to Ψ_+, and in agreement with Equation (16), the same Coulombic interaction between the matter nucleus and the electron, when the electron is in the state Ψ_-, cannot localize the electron wavefunction Ψ_-, and thus Ψ_a, around the nucleus of the Hydrogen-like matter atom. This is because no balance can be reached between the negative potential energy associated to $-eU_C\Psi_-$ in Equation (16), and the negative kinetic energy associated to:

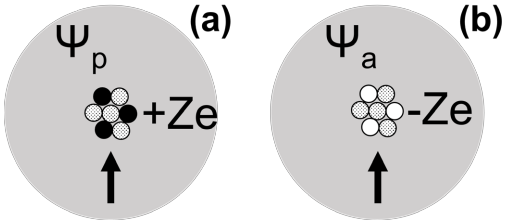

Figure 1. Illustration of (a) a matter and (b) antimatter Hydrogen-like atom. The arrows represent the centripetal acceleration of the electron and positron, respectively.

$$\hat{K}_{-}\Psi_{-}=\frac{\hat{p}^{2}}{2\mu_{-}}\Psi_{-}=-\frac{\hbar^{2}}{2\mu_{-}}\nabla^{2}\Psi_{-},\ \text{with}\ \mu_{-}=-\frac{1+\gamma}{2}m<0. \tag{23}$$

As it was stated above, the force associated to the nucleus-electron interaction points to the nucleus of the Hydrogen-like atom; however, when the electron is in the state Ψ_{-}, due to the negative effective mass in Equations (16) and (23) ($\mu_{-} < 0$), the same force produces an outward acceleration on the electron that tends to spatially spread Ψ_{-}, and thus Ψ_{a}. The kinetic energy of the electron in the state Ψ_{-} is negative because $\mu_{-} < 0$, thus large negative values of K_{-} imply large speed values. Consequently, the negative kinetic energy in Equations (16) and (23) still tends to spatially spread Ψ_{-}; therefore, it cannot balance the spreading effect of the nucleus-electron interaction on the wavefunction, when the electron is in the state Ψ_{-}.

If the matter nucleus of the Hydrogen-like atom were substituted by the nucleus of a Hydrogen-like antimatter atom, then the total charge producing the Coulomb potential would change from $+Ze$ to $-Ze$; therefore, Equations (15) and (16) should be substituted by the following equations:

$$i\hbar\frac{\partial}{\partial t}\Psi_{+}=-\frac{\hbar^{2}}{(\gamma+1)m}\nabla^{2}\Psi_{+}+eU_{C}\Psi_{+}. \tag{24}$$

$$i\hbar\frac{\partial}{\partial t}\Psi_{-}=\frac{\hbar^{2}}{(\gamma+1)m}\nabla^{2}\Psi_{-}+eU_{C}\Psi_{-}. \tag{25}$$

Now, in the antimatter atom, $-U_{C}$ cannot spatially localize the wavefunction of the electron ($\Psi_{p} = \Psi_{+}$) around the antimatter nucleus. In agreement with Equation (24), this would happen because the positive potential energy associated to $+eUC\Psi_{+}$ in Equation (24) cannot balance the positive kinetic energy associated to Equation (22). The force associated to the interaction between the antimatter nucleus and the electron, points in this case away from the antimatter nucleus to the electron; therefore, due to the positive effective mass of the electron in the state Ψ_{+} ($\mu_{+} > 0$), this force produces an outward acceleration on the electron that tends to spatially spread Ψ_{+}. Of course, the kinetic energy of the electron also tends to spread Ψ_{+}.

As illustrated in **Figure 1(b)**, in contrast with what occurs to the electron wavefunction ($\Psi_{p} = \Psi_{+}$) in a Hydrogen-like antimatter atom, and in agreement

with Equation (25), the same interaction between the electron and the antimatter nucleus could localize the wavefunction Ψ_-, and thus, due to Equation (21), the wavefunction of the positron (Ψ_a), around the nucleus of a Hydrogen-like antimatter atom. This is because the positive potential energy associated to $+eU_C\Psi_-$ in Equation (25) could balance the negative kinetic energy associated to Equation (23). The force associated to the interaction points away from the antimatter nucleus to the electron; however, when the electron in the antimatter atom is in the state $\Psi-$, due to the negative effective mass in Equation (25) ($\mu_- < 0$), the force produces an inward acceleration on the electron (**Figure 1(b)**) that tends to localize Ψ_-, and thus Ψ_a. As stated above, the negative kinetic energy in Equations (25) and (23) tends to spatially spread Ψ_-; therefore, it can balance the localizing effect of the anti-nucleus-electron interaction, when the electron is in the state Ψ_-.

It is worth noting that, in the Hydrogen-like atom, U_C can localize de electronic wavefunction Ψ_+ but not Ψ_-, while in the Hydrogen-like antiatom $-U_C$ can localize de electronic wavefunction Ψ_- but not Ψ_+. Consequently, in agreement with Equation (21), U_C can localize de particle wavefunction (Ψ_p) but not the antiparticle one (Ψ_a), while $-U_C$ can localize de antiparticle wavefunction (Ψ_a) but not the particle one (Ψ_p). U_C is charge-inversion-antisymmetric because U_C changes of sign when a C-inversion is applied to the external world surrounding the particle. In addition, the change of sign of U_C is independent of the change of sign of the particle that is moving through U_C.

Summarizing the above discussion, we can say the C-symmetry of Equations (13) and (14) indicates that a Hydrogen-like antimatter atom should be as stable as a Hydrogen-like matter atom. If all relevant potentials were of the form $V = \pm eU_C$, with U_C being charge-inversion-antisymmetric, we could confirm and extend the above statement to the whole matter and antimatter. However, as we will discuss in the next Section, we could conceive some relevant potentials that do not have this form.

4. Spatial Localization of the Wavefunction of a Single Free Particle or Antiparticle

Gravity potentials are not of the form $V = \pm eU_C$; however, gravity interactions between quantum particles are often weak when compared with electromagnetic interactions. Consequently, we could be tempted to discard the possible influence of gravity potentials on the observed asymmetry between matter and antimatter. However, at least a Newtonian-gravity potential has been previously suggested, as playing an important role in quantum physics [5] [16]. It has been hypothesized that a single free particle could interact gravitationally with itself, due to the spread of its mass density through the extension of its wavefunction [5] [16]. A consequence of this hypothesis, combined with Equation (13), is that no elemental quantum particle with $m > m_P$ could exist (in the state Ψ_+) [5]. The gravitational self-interaction potential does not depend on the particle's charge,

and the particle and the antiparticle have the same mass; therefore, one should expect that a similar requirement for existence cannot be obtained using Equation (14) for a particle in the state Ψ_-. Indeed, this is the case. From Equation (1), the energy of a particle, which wavefunction, Ψ_+ or Ψ_-, is localized in a finite space region with radius r, could be estimated using the following equation [5]:

$$E'_{\pm}(r)=\frac{\hbar^2}{\mu_{\pm}r^2}-\frac{Gm^2}{r},\ \text{with}\ \mu_+=\frac{1+\gamma}{2}m>0\ \text{and}\ \mu_-=-\frac{1+\gamma}{2}m<0. \quad (26)$$

In Equation (26), G is the gravitational constant. The "+" case in Equation (26) was used for obtaining the impossibility of the existence of quantum particles with $m > m_P$ (in the state Ψ_+) [5]. This is because $r \to 0$ when $m \to m_P$; *i.e.*, the quantum field (Ψ_+) "collapse" when $m = m_P$ [5]. In contrast to $E'_+(r)$, $E'_-(r)$ does not have a local extreme because, first, both the kinetic and potential energies in Equation (26) are negatives when the particle is in the state Ψ_-. Second, as illustrated in **Figure 2(b)**, the gravitational self-interaction force points to the "center" of the wavefunction Ψ_-; however, due to the negative effective mass in Equations (14) and (23) ($\mu_- < 0$), the gravitational force produces an outward acceleration on the particle in the state Ψ_- that tends to spatially spread Ψ_-. For this reason, when the particle is in the state Ψ_-, the gravitational force cannot balance the tendency of the kinetic energy to spread Ψ_-. This is opposite to what happens for a particle in the state $\Psi_p = \Psi_+$ (Equation (13)). For a particle (in the state Ψ_+), as illustrated in **Figure 2(a)**, the same force produces an inward acceleration that tends to localize Ψ_+ [5]. Consequently, when the particle is in the state Ψ_+, the gravitational force can balance the tendency of the kinetic energy to spatially spread Ψ_+.

Elemental quantum particles and antiparticles are created in pairs [3] [17] [21]; thus, an occupied particle state Ψ_+ is always created forming a pair with an unoccupied particle state Ψ_- [3] [17] [21]. Consequently, the impossibility of the existence of elemental particles (in the state Ψ_+) with $m > m_P$ is a sufficient condition for the impossibility of the existence of elemental particles in the state Ψ_- with $m > m_P$. *I.e.*, neither elemental quantum particles nor antiparticles should exist with $m > m_P$. Indeed, this is the case.

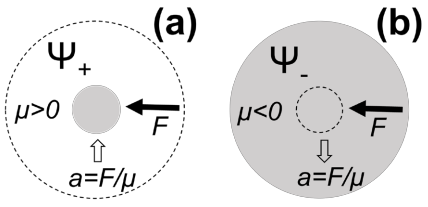

Figure 2. Illustration of the spatial (a) localization and (b) spread of the particle wavefunctions Ψ_+ and Ψ_-, respectively, due to gravitational self-interaction. Solid arrows represent the gravitational force and hollow arrows represent the corresponding acceleration. The discontinuous-line circle indicates the "initial size" of the wavefunctions.

The situation is different for the matter bodies with $m > m_P$ that surround us. These macroscopic objects are formed by numerous quantum particles with mass $m_i < m_P$, $i = 1, 2, \ldots$. It has been suggested that in this case, the collapse of the quantum field (Ψ_+), when $m > m_P$, means that these macroscopic objects are really what they look like: classical bodies [5]. Note that macroscopic matter and antimatter objects are not created in pairs; therefore, if macroscopic antimatter objects formed by numerous elemental antiparticles would exist, then from Equation (26) follows they could be massive quantum antimatter objects. This is because no collapse of the corresponding quantum field would occur no matter how massive the antimatter object is.

Self-interaction Newtonian-gravitation and electrostatic potentials are very similar but with opposite signs. The first is attractive, the second is repulsive. This could tempt anyone to explore the consequences of assuming that a single, free, and charged particle could interact electrostatically with itself, due to the spread of its charge density through the extension of its wavefunction. Certainly, this could be considered a controversial hypothesis. However, as it is shown below, the consequences of the electrical self-interaction hypothesis merit the investigation.

As for the gravitational self-interaction, using the electrostatic self-interaction hypothesis in combination with Equations (1), (13), and (14), we obtain that the "size" of a single, free, and charged (with a charge q) particle in the states Ψ_+ and Ψ_- should be the value of r corresponding to a local extreme of:

$$E'_{\pm}(r) = \frac{\hbar^2}{\mu_{\pm} r^2} + \frac{q^2}{4\pi\varepsilon_0 r}, \text{ with } \mu_+ = \frac{1+\gamma}{2} m > 0 \text{ and } \mu_- = -\frac{1+\gamma}{2} m < 0. \quad (27)$$

The second term in Equation (27) corresponds to the positive potential energy associated to the repulsive electrostatic self-interaction of a particle with itself. Clearly, only $E'_-(r)$ can have a local extreme. This is because the potential and kinetic energies in Equation (27) have different signs if and only if the particle is in the state Ψ_-. As illustrated in **Figure 3**, the force associated with the electrostatic self-interaction points away from the "center" of the particle wavefunction. As illustrated in **Figure 3(a)**, this force produces an outward acceleration on the particle that tends to spread Ψ_+ (Equations (13) and (27) with $\mu_+ > 0$), but an inward acceleration (**Figure 3(b)**) that tends to localize Ψ_- (Equations (14) and (27) with $\mu_- < 0$). The kinetic energy always tends to spread the wavefunction; consequently, only Ψ_- can be localized by the electrostatic self-interaction. Note that, as this will be discussed in Section 6, the potential responsible of the electrostatic self-interaction is not of the form $V = \pm qU_C$. Equaling to zero the derivative of $E'_-(r)$ respect to r, and solving respect to r the resulting equation, we found $E'_-(r)$ has a local maximum when [23]:

$$r = \lambda\!\!\!^{-}_C \xi^{-2} \sqrt{1-\xi^4}, \text{ with } \xi = \frac{q}{q_P}, \; q_P = \sqrt{4\pi\epsilon_0 \hbar c}, \text{ and } \lambda\!\!\!^{-}_C = \frac{\hbar}{mc}. \quad (28)$$

Thus $\lambda\!\!\!^{-}_C \xi^2 \to \lambda\!\!\!^{-}_C$ (the reduced Compton wavelength) when $|q| \to q_P$. Moreover, $r \to 0$, *i.e.*, the quantum field Ψ_- and thus Ψ_a collapse when $|q| \to q_P$. This could

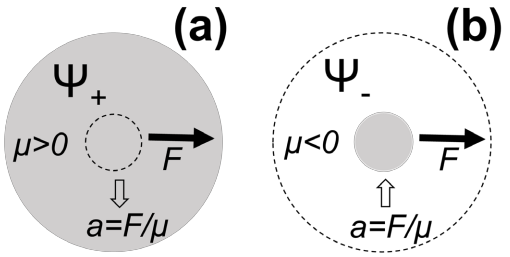

Figure 3. Illustration of the spatial (a) spread and (b) localization of the particle wavefunctions Ψ_+ and Ψ_-, respectively, due to electrostatic self-interaction. Solid arrows represent the electrostatic force and hollow arrows represent the corresponding acceleration. The discontinuous-line circle indicates the "initial size" of the wavefunctions.

be interpreted as the impossibility of the existence of quantum antiparticles with $|q|$ larger than the Plank charge ($q_P \approx 11e$). Indeed, there are no known elemental antiparticles with $|q| > q_P$. This also means that there cannot be elemental particles with $|q| > q_P$ because particles and antiparticles have the same values of m and $|q|$, and they are created simultaneously in pairs.

Following similar arguments than the used for the prediction of the existence of primordial black holes with $m > m_P$ [5] [24], we could now speculate about what could have happened to the elemental free antiparticles with $|q| > q_P$. Or they never existed or, if they existed, they electrically collapsed to a kind of primordial electrical sinks, which are hypothesized for the first time here. If there was a time when no elemental charged antiparticles existed, the primordial charge density fluctuations with $|q| < q_P$ evolved in the elemental charged particles and antiparticles existing today. However, the primordial charge density fluctuations of antimatter with $|q| > q_P$ could have evolved in primordial electrical sinks. Interestingly, if such primordial electrical sinks formed of charged antimatter existed or exist; their existence would imply the existence of an excess of charged matter in the rest of the Universe.

5. Absence of Antimatter with a Complicated Structure in the Known Universe

Equations (27) and (28) apply to any quantum object with mass and charge. The quantum field (Ψ_-), and thus, due to Equation (21), the antimatter quantum field (Ψ_a), both collapse when $|q| > q_P$. This collapse could mean that charged antimatter objects, formed by numerous elemental antiparticles with $|q_i| < q_P$, $i = 1, 2, \ldots$ should be classical antimatter bodies. Note that large matter and antimatter objects are not created in pairs; therefore, this conclusion does not apply to existing charged matter object surrounding us.

Now we can see a possible theoretical explanation for the absence of antimatter with a complicated structure in the known Universe. The existence of molecules,

gases, rocks, planets, oceans, and live in the known Universe requires the existence of atoms. Stable atoms require the interaction of a quantum nucleus with a quantum electron cloud. However, an antimatter nucleus containing more than 11 antiprotons and a cloud containing more than 11 positrons should be classical. Consequently, only very light antimatter atoms should be stable. Indeed, this is both an experimental and an everyday fact [25] [26].

Finally, we should explore the possibility that a charged particle could interact with itself both gravitationally and electrically. This can be accounted for by modifying Equation (27) in the following way:

$$E'_{\pm}(r)=\frac{\hbar^2}{\mu_{\pm}r^2}+\frac{\frac{q^2}{4\pi\epsilon_0}-m^2G}{r},\ \text{with}\ \mu_{+}=\frac{1+\gamma}{2}m>0\ \text{and}\ \mu_{-}=-\frac{1+\gamma}{2}m<0. \quad (29)$$

Therefore, Ψ_{+} can be spatially localized if:

$$\left(\frac{q}{m}\right)^2<4\pi\epsilon_0 G=\left(\frac{q_P}{m_P}\right)^2\approx 7.4\times10^{-21}\frac{\mathrm{C}^2}{\mathrm{kg}^2}. \quad (30)$$

While Ψ_{-} can be spatially localized if:

$$\left(\frac{q}{m}\right)^2>\left(\frac{q_P}{m_P}\right)^2. \quad (31)$$

Note that Equation (29) reduces to Equation (27) when $|q|/m \gg q_P/m_P$. This is the case, for instance, for positrons and antiprotons, that have values of $|q|/m \approx 3.1\times10^{22}$ and 9.2×10^{15} $\mathrm{C}^2/\mathrm{kg}^2$, respectively. For any neutral particle like the neutron, $|q|/m \ll q_P/m_P$, therefore, Equation (29) reduces to Equation (26). From Equation (29) follows that hypothetical original antimatter fluctuations of mass and charge densities such that $|q|/m \gg q_P/m_P$ and $|q| > q_P$, and original matter fluctuations of mass and charge densities with $|q|/m \ll q_P/m_P$ and $m > m_P$ could have collapsed in primordial electrical sinks and gravitational black holes, respectively.

6. Discussion

In the previous two Sections was shown that the hypothesis, of a free charged particle interacting electrostatically with itself, seems to imply an asymmetry between matter and antimatter like the one indeed existing in Nature. If a free particle of charge q could interact electrostatically with itself, Equations (13) and (14) should be modified in the following way [16]:

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\Psi_p&=-\frac{\hbar^2}{(\gamma+1)m}\nabla^2\Psi_p+V_{si+}\Psi_p,\\ V_{si+}&=\frac{q^2}{4\pi\varepsilon_0}\int\frac{\left|\Psi_{+}(r',t)\right|^2}{|r'-r|}\mathrm{d}V'>0,\ \Psi_p=\Psi_{+}. \end{aligned} \quad (32)$$

$$i\hbar\frac{\partial}{\partial t}\Psi_{-}=\frac{\hbar^2}{(\gamma+1)m}\nabla^2\Psi_{-}+V_{si-}\Psi_{-},\ V_{si-}=\frac{q^2}{4\pi\varepsilon_0}\int\frac{\left|\Psi_{-}(r',t)\right|^2}{|r'-r|}\mathrm{d}V'>0. \quad (33)$$

Note that the potential responsible of the electrostatic self-interaction is not of the form $V = \pm qU_C$. This is because $q/(4\pi\varepsilon_o r)$ is not charge-inversion-antisymmetric, *i.e.*, it does not change of sign when a C-inversion is applied to the external world surrounding the particle.. Also, unlike U_C, $q/(4\pi\varepsilon_o r)$ does change of sign when the particle changes of sign. As it was discussed in the previous section, V_{si+} in Equation (32) tends to spatially spread $\Psi_p = \Psi_+$. However, the repulsive electrostatic self-interaction tends to spatially shrink Ψ_-. Consequently, for the corresponding free antiparticle interacting electrostatically with itself, if Equation (21) should remain valid, Equation (17) should be modified in the following way:

$$i\hbar\frac{\partial}{\partial t}\Psi_a = -\frac{\hbar^2}{(\gamma+1)m}\nabla^2\Psi_a - V_{si-}\Psi_a. \tag{34}$$

So that Equation (34) can be obtained from Equation (33) as Equation (17) can be obtained from Equation (16); *i.e.*, by taking the complex conjugate of both sides of Equation (33). Consequently, the antiparticle should interact with itself in a different way that the charged particle does. Equation (34) differs from Equation (32) in that $-V_{si-}$ does not tend to spatially spread Ψ_a but, like it does to Ψ_-, $-V_{si-}$ tends to shrink Ψ_a. The discussion, presented in the two previous Sections, is consistent with choosing Equation (34) as the correct equation for the hypothetical self-interacting free antiparticle. This implies that particles interact differently with itself than antiparticles, thus introducing a fundamental asymmetry between elemental charged particles and antiparticles.

From Equation (27) follows that the wavefunction of a free electron or a free proton should be an extended plane wave. However, by evaluating Equation (28) for a positron and an antiproton, we obtain the wavefunction of a free positron should be confined in a spatial region of radius equal to the Bohr radius ($r \approx 0.05$ nm), while the wavefunction of a free antiproton should be confined in a region of radius $r \approx 29$ fm. This may be verifiable predictions of the correctness of Equation (34). Experiments comparing the spectra of matter and antimatter Hydrogen atoms have been reported and continue being conducted [27] [28]. This may provide another possibility for experimentally checking the hypothesis that a single particle could interact with itself, due to the spread of its charge density through the extension of its wavefunction. If the electron could interact electrostatically with itself, Equations (15) and (17) should be modified in the following way:

$$i\hbar\frac{\partial}{\partial t}\Psi_p = -\frac{\hbar^2}{(\gamma+1)m}\nabla^2\Psi_p - \left(eU_C - V_{si+}\right)\Psi_p. \tag{35}$$

$$i\hbar\frac{\partial}{\partial t}\Psi_a = -\frac{\hbar^2}{(\gamma+1)m}\nabla^2\Psi_a - \left(eU_C + V_{si-}\right)\Psi_a. \tag{36}$$

The self-interaction term in Equation (35) is not included in a description of a matter Hydrogen-like atom using the Klein-Gordon or Dirac equations [3] [17].

V_{si+} tends to spatially spread Ψ_p, due to the screening of U_C by the electron wavefunction. This is in reminiscence of the Lamb shift, which is explained, in a quantum field theory context, as produced by the screening of U_C by the random fluctuations of the vacuum, thus lasting producing a slight spatially spreading of Ψ_p [29] [30] [31]. In contrast, $-V_{si-}$ tends to spatially shrink Ψ_a. Consequently, a small difference between the precise energy level structure of the matter and antimatter Hydrogen atoms should exist. However, if these subtle differences were experimentally discarded, this would not imply that particles could not interact with itself. Such negative experimental result could just mean that Equation (34) should be substituted by:

$$i\hbar\frac{\partial}{\partial t}\Psi_a = -\frac{\hbar^2}{(\gamma+1)m}\nabla^2\Psi_a + V_{si-}\Psi_a. \tag{37}$$

So that Equation (37) can be obtained from Equation (33) by changing the sign of the kinetic energy term in Equation (33) without changing the sign of the potential term. This is not equivalent to taking the complex conjugate of both sides of Equation (33).

However, if the correct equation for a free self-interacting antiparticle were not Equation (34) but Equation (37), then Equation (37) could still be obtained from Equation (33) by changing the sign of the kinetic energy term in Equation (33), and then changing the sign of the particle's charge. This is because in contrast to $-eU_C$ in Equation (16), V_{si-} is proportional to the square of the particle's charge. Also, in contrast to $-eU_C$ in Equation (17), V_{si-} in Equation (37) does not change of sign if the matter nucleus is substituted by the corresponding antimatter one. Consequently, if the correct equation for a free self-interacting antiparticle were not Equation (34) but Equation (37), then particles and antiparticles would interact equally with itself, and matter and antimatter would continue being theoretically symmetric. However, Equation (21) would not be valid for self-interacting potentials. Adopting Equation (37) as the correct antiparticle equation could be justified because V_{si+} and V_{si-} are not, strictly speaking, scalar potentials due to their dependence on Ψ_+ and Ψ_-. Consequently, Equations (32) and (33) do not strictly follow from Equations (13) and (14) but they really are an *ansatz*.

It should also be noted that the spin of the particle has not been included in the previous discussions. This is because the kinetic energy of a particle, which is in the ground state and spatially confined in a cube of size r, does not dependent of the particle's spin and is proportional to $\hbar^2/\mu r^2$ [3] [5] [6] [11], which is the kinetic energy expression used in Equations (26), (27), and (29).

7. Conclusions

A pair of decoupled Schrödinger-like, but relativistic quantum mechanics gGP equations were explored. One, with effective mass $\mu_+ > 0$, directly describes a single relativistic quantum particle in a quantum state with $K > 0$. The other, with $\mu_- < 0$, indirectly describes a single relativistic quantum antiparticle in a quantum

state with $K > 0$. The simplicity and the independence of these equations facilitated the study of the spatial localization of the wavefunctions of particles and antiparticles.

It was studied, for the first time, a gGP equation with a negative effective mass value (Equation (1) with $\mu_- < 0$). The solutions of this equation are the wavefunctions Ψ_-, which are related to the antiparticle's wavefunctions (Ψ_a) through Equations (20) and (21). The energy values corresponding to Ψ_- are $E'_- = -E'_a$, where E'_a is the energy of the associated antiparticle in a quantum state with $K > 0$.

It was found that the antiparticle's wavefunction corresponding to a quantum state with $K > 0$, Ψ_a, can be spatially localized by a scalar potential of the form $V = \pm eU_C$ if, and only if, the related particle's wavefunction corresponding to a quantum state with $K < 0$, Ψ_-, can be spatially localized (Equation (21)).

As expected, it was found that matter and antimatter are indistinguishable if U_C in Equations (15), (16), and (17) is Coulombic and produced by the external world where the particle or antiparticle exists. When this happens, U_C change of sign under a C-transformation is applied to the external world where the particle exists, but it does not change of sign when the sign of the particle's charge changes. It was shown that in this case, both gGP equations are C-symmetric, and matter and antimatter are undistinguishable.

However, if the potential is not produced by the external world surrounding the particle (or antiparticle) but by the interaction of the particle (or antiparticle) with itself, and Equations (21) and (34) are valid, then matter and antimatter could be distinguished because both gGP equations are not C-symmetric.

Finally, it was shown that if a quantum particle (or antiparticle) could interact with itself, due to the spread of the charge and mass densities through its wavefunction, and Equations (21) and (34) are valid, then there should not be elemental particles and antiparticles with $m > m_P$ or $|q| > q_P$. It was proposed the possible existence of primordial antimatter electrical sinks. These hypothetical antimatter objects could be partially responsible for the observed excess of charged matter in Nature. It was also suggested that all macroscopic matter bodies such that $|q|/m \ll q_P/m_P$ and $m > m_P$, and all macroscopic antimatter bodies such that $|q|/m \gg q_P/m_P$ and $|q| > q_P$, should be classical objects. This could explain the absence of antimatter with a complicated structure in the known Universe.

The author is aware that the results presented in this work could be used in two opposite ways. Those convinced that elemental particles are mathematical points, and convinced of the universality of the CPT symmetry, could argue that this work reinforces their belief. Others, that are perplexed by the fact that in contrast with these beliefs, Nature seems to be mostly made of matter, could argue that this work points to a plausible explanation of why Nature is as it seems to be.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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